

**REGRESSION ANALYSIS IN  
STATISTICS**

**Presented**

**By**

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# OUTLINE

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- ✓ *Introduction*
- ✓ *Concepts of Regression*
- ✓ *Assumptions*
- ✓ *Simple Bivariate Analysis:*
- ✓ *Fitting of Regression Trend Line by  
Least Square Method.*
- ✓ *Residual Mapping.*

# *INTRODUCTION*

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## What is regression?

- A reliable method of identifying which variables have impact on a topic of interest.
- We can determine which factors matter most
- Which factors to be ignored
- How these factors influence each other

# IMPORTANT TERMS IN REGRESSION

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- **Dependent Variable:** The main factor that you're trying to understand or predict.
- **Independent Variable:** These are the factors that you hypothesize, show an impact on the dependent variable
- **Coefficients:** They represent the change in the response variable for a unit change in predictor variable
- **Error term:** The difference between the actual and predicted values

# GENERAL EQUATION FOR REGRESSION

- The simple linear regression equation is

$$y = \beta_0 + \beta_1 x_1$$

- The Multiple linear regression equation is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

- Coefficient is computed by subtracting mean from the variable and dividing by the standard deviation
- Error sum of squares is calculated by  $\varepsilon_i = y_i - \hat{y}_i$

# LEAST SQUARES METHOD

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- ✓ The Least Squares method is a form of mathematical regression analysis used to determine the line of best fit for a set of data,
- ✓ It aims to create a straight line that minimizes the sum of the squares of the errors
- ✓ It is mostly used for data fitting
- ✓ Our aim is to minimise the sum of squares

*Fitting of Regression Trend Line by  
Least Square Method*

|   |    |    |    |    |    |     |     |    |
|---|----|----|----|----|----|-----|-----|----|
| Average Monthly Temperature ( <sup>0</sup> C) | 38 | 29 | 25 | 21 | 34 | 41  | 43  | 28 |
| Electricity Bill (Rs.)                        | 85 | 75 | 60 | 55 | 90 | 100 | 120 | 70 |

On the basis of the data

- i) Draw a Regression Line following the least square method.
- ii) Predict the probable <sup>0</sup>C in average monthly temperature of rupees in electricity bill is 80.
- iii) Predict the Probable rupees in electricity bill of <sup>0</sup>C, when average monthly temperature is 40<sup>0</sup> C.
- iv) Interpret the relationship.

| Average Monthly Temperature ( <sup>0</sup> C) (x) | Electricity Bill (Rs.) (y) | x.y                  | X <sup>2</sup>      |
|---|----------------------------|----------------------|---------------------|
| 38  | 85                         | 3230                 | 1444                |
| 29  | 75                         | 2175                 | 841                 |
| 25  | 60                         | 1500                 | 625                 |
| 21  | 55                         | 1155                 | 441                 |
| 34  | 90                         | 3060                 | 1156                |
| 41  | 100                        | 4100                 | 1681                |
| 43  | 120                        | 5160                 | 1849                |
| 28  | 70                         | 1960                 | 784                 |
| $\Sigma x = 259$                                  | $\Sigma y = 655$           | $\Sigma x.y = 22340$ | $\Sigma X^2 = 8821$ |

It is known as the method of least square. The general equation of a straight line is given by  $y = a + bx$ , where **a = intercept**, **b= slope**. The equation to solve for a and b are

$$\Sigma y = na + b \Sigma x \dots\dots\dots (i)$$

$$\Sigma xy = a \Sigma x + b \Sigma x^2 \dots\dots\dots (ii)$$

By substituting the values from the table we

get,  $655 = 8a + 259b \dots\dots\dots (i)$

$22340 = 259a + 8821b \dots\dots\dots (ii)$

**Multiplying equation (i) by 32.38 we get,**



$$21208.90 = 259a + 8386.42b \dots\dots\dots (iii)$$

$$22340 = 259259a + 21b \dots\dots\dots (ii)$$

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$$-1131.10 = -434.58b$$

$$\text{Or } 434.58b = 1131.10$$

$$\text{Or, } b = 2.60$$

Substituting the value of b in eq (i) we get,

$$655 = 8a + 259b$$

$$\text{Or, } 655 = 8a + 259 * 2.60$$

$$\text{Or, } 655 = 8a + 665.63$$

$$\text{Or, } -8a = 665.40 - 655$$

$$\text{Or, } -8a = 10.63$$

$$\text{Or, } a = 10.63 / -8$$

$$\text{Or, } a = -1.33$$

$$y = a + bx$$

$$y = -1.33 + 2.60x$$

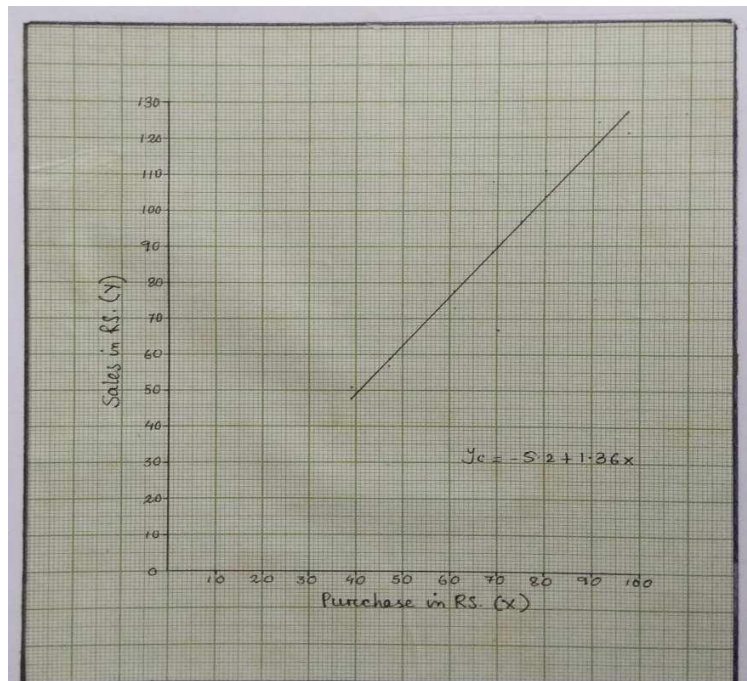
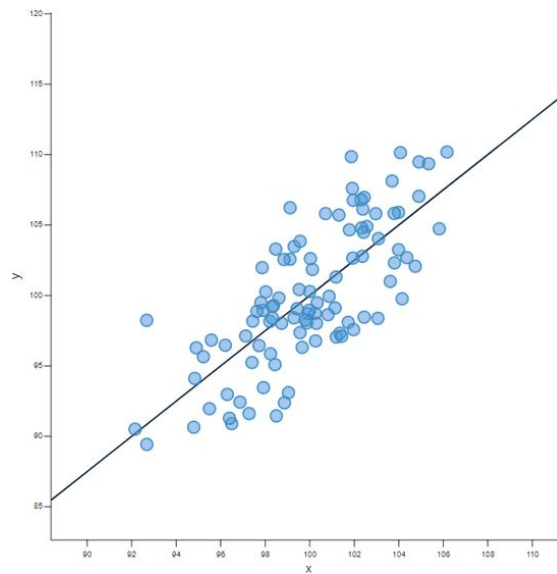
**Therefore, Regression Equation:**

$$y = -1.33 + 2.60x$$

| x  | y <sub>c</sub> |
|----|----------------|
| 38 | 47.12          |
| 29 | 36.83          |
| 25 |                |
| 21 |                |
| 34 |                |
| 41 |                |
| 43 |                |
| 28 |                |

The Slope of the Regression line 'b', which is also known as regression co-efficient, shows the estimated average change in y with respect to x. Thus from the above regression equation, we can say that the relationship between **Average Monthly Temperature** and **Electricity Bill (Rs.)** is such that a  $^{\circ}\text{C}$  increase in temperature by and large cause an increase of 2.60 Rs. in electricity bill.

### SCATTER DIAGRAM AND REGRESSION LINE OF Y ON X



# MEAN SQUARE ERROR

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- ✓ The mean squared error tells you how close a regression line is to a set of points.
- ✓ It does this by taking the distances from the points to the regression line
- ✓ The squaring is necessary to remove any negative signs.
- ✓ Formula for Mean square error is  $E = \text{Sum}(Y - \hat{Y})^2$

# *Residual Mapping*

| Sl. No. | Name of the District | Percentage of Urban Population (y) | Percentage of total worker (x) |
|---------|----------------------|------------------------------------|--------------------------------|
| 1       | Patna                | 22.54                              | 3.16                           |
| 2       | Gaya                 | 7.63                               | 1.40                           |
| 3       | Sahabad              | 8.22                               | 2.63                           |
| 4       | Serdan               | 4.20                               | 2.24                           |
| 5       | Champanan            | 5.21                               | 1.47                           |
| 6       | Muzaffarpur          | 5.25                               | 1.33                           |
| 7       | Darbhangha           | 4.43                               | 1.45                           |
| 8       | Monghyr              | 11.87                              | 3.16                           |
| 9       | Bhagalpur            | 10.61                              | 2.00                           |
| 10      | Saharsa              | 4.53                               | 0.58                           |
| 11      | Purnea               | 6.34                               | 1.25                           |
| 12      | Santal Pargana       | 5.76                               | 1.55                           |
| 13      | Palamau              | 4.96                               | 1.20                           |
| 14      | Hazaribagh           | 12.87                              | 1.93                           |
| 15      | Ranchi               | 13.67                              | 4.13                           |
| 16      | Dhanbad              | 43.51                              | 8.95                           |
| 17      | Sighbhum             | 26.24                              | 11.84                          |

- (i) Find out the regression equation of y on x using the values given above (1971).
- (ii) What is the average rate of change y in with respect to x
- (iii) Further calculate residual and plot the residual over map.

| Name of the District | Percentage of Urban Population (y) | Percentage of total worker (x) | xy     | $x^2$ | $Y_c$ |
|----------------------|------------------------------------|--------------------------------|--------|-------|-------|
| Patna                | 22.54                              | 3.16                           | 71.23  | 9.99  |       |
| Gaya                 | 7.63                               | 1.40                           | 10.68  | 1.96  |       |
| Sahabad              | 8.22                               | 2.63                           | 21.62  | 6.92  |       |
| Serdan               | 4.20                               | 2.24                           | 9.41   | 5.02  |       |
| Champanan            | 5.21                               | 1.47                           | 7.66   | 2.16  |       |
| Muzaffarpur          | 5.25                               | 1.33                           | 6.98   | 1.77  |       |
| Darbhangha           | 4.43                               | 1.45                           | 6.42   | 2.10  |       |
| Monghyr              | 11.87                              | 3.16                           | 37.51  | 9.99  |       |
| Bhagalpur            | 10.61                              | 2.00                           | 21.22  | 4     |       |
| Saharsa              | 4.53                               | 0.58                           | 2.63   | 0.34  |       |
| Purnea               | 6.34                               | 1.25                           | 7.93   | 1.56  |       |
| Santal Pargana       | 5.76                               | 1.55                           | 8.93   | 2.40  |       |
| Palamau              | 4.96                               | 1.20                           | 5.95   | 1.44  |       |
| Hazaribagh           | 12.87                              | 1.93                           | 24.84  | 3.72  |       |
| Ranchi               | 13.67                              | 4.13                           | 56.46  | 17.06 |       |
| Dhanbad              | 43.51                              | 8.95                           | 389.41 | 80.10 |       |

|          |                        |                       |                         |                          |  |
|----------|------------------------|-----------------------|-------------------------|--------------------------|--|
| Sighbhum | 26.24                  | 11.84                 | 310.68                  | 140.19                   |  |
|          | $\Sigma y =$<br>197.66 | $\Sigma x =$<br>50.27 | $\Sigma x.y$<br>=999.56 | $\Sigma X^2 =$<br>290.72 |  |

$$\Sigma y = na + b \Sigma x \dots\dots\dots (i)$$

$$\Sigma xy = a \Sigma x + b \Sigma x^2 \dots\dots\dots (ii)$$

$$197.66 = 17a + 50.27 b \dots\dots\dots (i)$$

$$22340 = 50.27a + 290.72 b \dots\dots\dots (ii)$$

Multiplying equation (i) by 2.96 we get,

$$585.07 = 50.27 a + 148.80 b \dots\dots\dots (iii)$$

$$999.24 = 50.27 a + 290.72 b \dots\dots\dots (ii)$$

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$$\begin{array}{r} -414.17 \qquad -141.92 b \\ = \end{array}$$

$$\text{Or } 141.92 b = 414.17$$

$$\text{Or, } b = 2.92$$

Substituting the value of b in eq (i) we get,

$$197.66 = 17a + 50.27 b$$

$$\text{Or, } 197.66 = 17 a + 50.27 * 2.92$$

$$\text{Or, } a = 2.99$$

(ii) Same as previous one

| Name of the District | Percentage of Urban Population (y) | Y hat | Residual (Y-Y hat) | (Y-Y hat) <sup>2</sup> | Categories      |
|----------------------|------------------------------------|-------|--------------------|------------------------|-----------------|
| Patna                | 22.54                              | 12.22 | 10.32              | 106.50                 | High Positive   |
| Gaya                 | 7.63                               | 7.08  | 0.55               | 0.30                   | High Positive   |
| Sahabad              | 8.22                               | 10.67 | -2.45              | 6.00                   | Medium Negative |
| Serdan               | 4.20                               | 9.53  | -5.33              | 28.41                  |                 |
| Champan              | 5.21                               | 7.28  | -2.07              | 4.28                   |                 |
| Muzaffarpur          | 5.25                               | 6.87  | -1.62              | 2.62                   |                 |
| Darbhanga            | 4.43                               | 7.22  | -2.79              | 7.78                   |                 |
| Monghyr              | 11.87                              | 12.22 | -0.35              | 0.12                   |                 |
| Bhagalpur            | 10.61                              | 8.83  | 1.78               | 3.17                   |                 |
| Saharsa              | 4.53                               | 4.68  | -0.15              | 0.02                   |                 |
| Purnea               | 6.34                               | 6.64  | -0.3               | 0.09                   |                 |
| Santal Pargana       | 5.76                               | 7.52  | -1.76              | 3.10                   |                 |
| Palamau              | 4.96                               | 6.49  | -1.8               | 3.24                   |                 |
| Hazaribagh           | 12.87                              | 8.63  | 4.25               | 18.06                  |                 |
| Ranchi               | 13.67                              | 15.05 | -1.38              | 1.90                   |                 |
| Dhanbad              | 43.51                              | 29.12 | 14.39              | 207.07                 |                 |
| Sighbhum             | 26.24                              | 37.56 | -11.32             | 128.14                 |                 |
|                      |                                    |       | $\Sigma = 0.03$    | $\Sigma = 520.8$       |                 |

**Mean of Residual =  $0.03/17 = 0.00$**

**SD of Estimate =  $\sqrt{520.80/17-2} = 5.89$**

### **Categories**

**0 to 5.89 = Medium Positive**

**5.89 to 11.78 = High Positive**

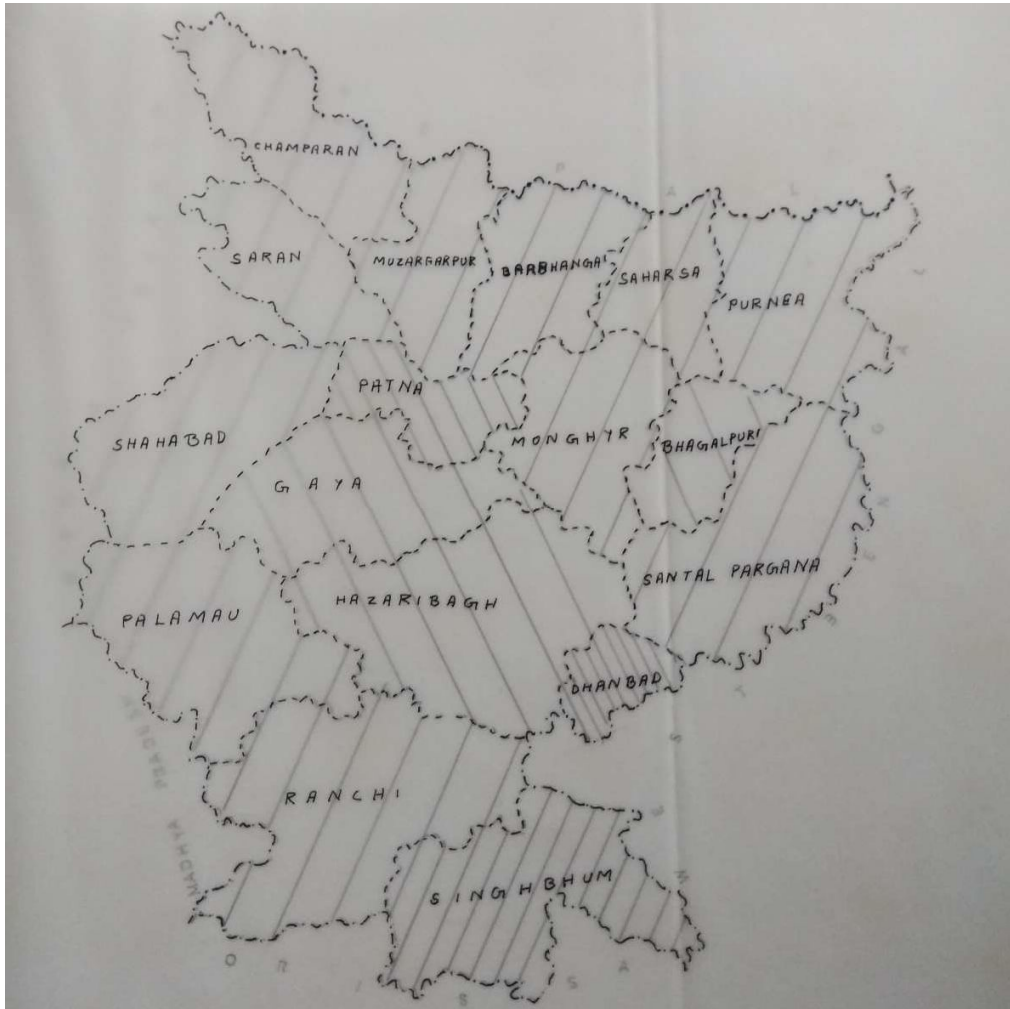
**> 11.78 = Very high Positive**

**0 to - 5.89 = Medium Negative**

**-5.89 to -11.78 = High Negative**

**> -11.78 = Very high Negative**

**Residuals from Regression of Urbanisation on Percentage of employment in Manufacturing Industries in Bihar (1971)**





***THANK YOU***

For you're attention

Please don't hesitate to ask, I will be  
always ready to help you